END SEM SOLUTIONS

1. $$n_0^{2D} = \int_{\mathcal{E}_c}^{\infty} g_{2D}(\mathcal{E})\, f(\mathcal{E})\, d\mathcal{E} = \int_{\mathcal{E}_c}^{\infty} \left(\frac{4\pi m_n^*}{h^2}\right) \frac{1}{1+e^{(\mathcal{E}-\mathcal{E}_F)\beta}}\, d\mathcal{E}$$

$$\simeq \frac{4\pi m_n^*}{h^2} \int_{\mathcal{E}_c}^{\infty} e^{-(\mathcal{E}-\mathcal{E}_F)\beta}\, d\mathcal{E}$$

Let, $(\mathcal{E}-\mathcal{E}_c)\beta = x$.

$\Rightarrow \beta d\mathcal{E} = dx$

$$= \frac{4\pi m_n^*}{h^2} e^{\mathcal{E}_F\beta} \int_0^{\infty} e^{-(x+\mathcal{E}_c\beta)} \frac{dx}{\beta}$$

$$= \frac{4\pi m_n^*}{h^2\beta} e^{(\mathcal{E}_F-\mathcal{E}_c)\beta}$$

$$\therefore\ n_0^{2D} = \frac{4\pi m_n^*}{h^2} k_B T\, e^{-(\mathcal{E}_c-\mathcal{E}_F)\beta} \quad \text{(i)}$$

$$p_0^{2D} = \int_{-\infty}^{\mathcal{E}_v} g_{2D}(\mathcal{E}) \{1-f(\mathcal{E})\}\, d\mathcal{E} = \frac{4\pi m_p^*}{h^2} \int_{-\infty}^{\mathcal{E}_v} \left\{1 - \frac{1}{1+e^{(\mathcal{E}-\mathcal{E}_F)\beta}}\right\} d\mathcal{E}$$

$$= \frac{4\pi m_p^*}{h^2} \int_{-\infty}^{\mathcal{E}_v} \frac{e^{(\mathcal{E}-\mathcal{E}_F)\beta}}{1+e^{(\mathcal{E}-\mathcal{E}_F)\beta}}\, d\mathcal{E}.$$

$$= \frac{4\pi m_p^*}{h^2} \int_{-\infty}^{\mathcal{E}_v} \frac{1}{e^{(\mathcal{E}_F-\mathcal{E})\beta}+1}\, d\mathcal{E}$$

Let, $(\mathcal{E}_v-\mathcal{E})\beta = x$.

$\Rightarrow -\beta d\mathcal{E} = dx$

$$\simeq \frac{4\pi m_p^*}{h^2} \int_{-\infty}^{\mathcal{E}_v} e^{-(\mathcal{E}_F-\mathcal{E})\beta}\, d\mathcal{E}$$

$$= \frac{4\pi m_p^*}{h^2} e^{-\mathcal{E}_F\beta} \int_{\infty}^{0} e^{(\mathcal{E}_v\beta - x)} \frac{dx}{-\beta}$$

$$= \frac{4\pi m_p^*}{h^2\beta} e^{-(\mathcal{E}_F-\mathcal{E}_v)\beta}$$

$$\therefore\ p_0^{2D} = \frac{4\pi m_p^*}{h^2} k_B T\, e^{-(\mathcal{E}_F-\mathcal{E}_v)\beta} \quad \text{(ii)}$$

For intrinsic semiconductor,

$$n_0^{2D} = p_0^{2D}$$

$$\frac{4\pi m_n^*}{h^2} k_B T\, e^{-(\mathcal{E}_c-\mathcal{E}_F)\beta} = \frac{4\pi m_p^*}{h^2} k_B T\, e^{-(\mathcal{E}_F-\mathcal{E}_v)\beta}$$

$$\Rightarrow \ln m_n^* - \frac{\mathcal{E}_c-\mathcal{E}_F}{k_B T} = \ln m_p^* - \frac{\mathcal{E}_F-\mathcal{E}_v}{k_B T}$$

$$\Rightarrow \frac{2\mathcal{E}_F - \mathcal{E}_c - \mathcal{E}_v}{k_B T} = \ln \frac{m_p^*}{m_n^*}$$

$$\Rightarrow \mathcal{E}_F - \frac{\mathcal{E}_c+\mathcal{E}_v}{2} = \frac{k_B T}{2} \ln \frac{m_p^*}{m_n^*} = \frac{25.875 \text{ meV}}{2} \ln(2)$$

$$\Rightarrow \boxed{\mathcal{E}_{F_i} - \text{midgap} \simeq 8.97 \text{ meV}}$$

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2. The curie temp is defined as : $T_c = \dfrac{g_J \mu_B (J+1) \lambda M_s}{3 k_B} = \dfrac{n \lambda \mu_{eff}^2}{3 k_B}$

$$\Rightarrow T_c = \frac{n \lambda g_J^2 \mu_B^2 J(J+1)}{3 k_B} \qquad \left[\mu_{eff} = g_J \mu_B \sqrt{J(J+1)}\right]$$

for BCC , $n = \dfrac{2}{(2.8665 \times 10^{-10})^3}$ & putting the other values in the above expression,

$$\Rightarrow 997.36 = \frac{\frac{2}{(2.8665\times10^{-10})^3} \times 100 \mu_0 \times (9.274\times10^{-24})^2}{3 \times 1.3806 \times 10^{-23}} \times g_J^2 J(J+1)$$

$$\Rightarrow 997.36 = \frac{\frac{2}{(2.8665\times10^{-10})^3} \times 100 \times 1.2566 \times 10^{-6} \times (9.274\times10^{-24})^2}{3 \times 1.3806 \times 10^{-23}} \times g_J^2 J(J+1)$$

$$\Rightarrow g_J J \cdot g_J (J+1) \simeq 45$$

$$\Rightarrow 6 \times (g_J J + g_J) = 45 \quad ; \because g_J J = 6$$

$$\Rightarrow 6 \times (6 + g_J) = 45$$

$$\Rightarrow g_J = \frac{3}{2} \longrightarrow \therefore J = 4$$

Now $J=4$ is possible for two configurations :

d^6	d^8
↓↑ | ↓ | ↓ | ↓ | ↓	↓↑ | ↓↑ | ↓↑ | ↓ | ↓
$S = 2$	$S = 1$
$L = 2$	$L = 3$
$J = 4$	$J = 4$

$$g_J = \frac{3}{2} + \frac{2(2+1) - 2(2+1)}{2 \times 4 (4+1)} = \frac{3}{2} \ ✓$$

$$g_J = \frac{3}{2} + \frac{1(1+1) - 3(3+1)}{2 \times 4 (4+1)} = \frac{3}{2} - \frac{10}{40} = \frac{5}{4} \ ✗$$

$\therefore$ The electronic configuration is d^6.

3. a) At 35K $N_D = 5\times10^{18}\ cm^{-3}$

$n_0 = 4.16\times10^{17}\ cm^{-3}$

$U_c = 8\times10^{16}\ cm^{-3}$

Now we know, $N_D = n_d + n \Rightarrow n_d = N_D - n = (5\times10^{18} - 4.16\times10^{17})\ cm^{-3}$

$= 4.584\times10^{18}\ cm^{-3}$

$$\frac{n_d}{N_D} = \frac{1}{1 + \frac{U_c}{2N_D} e^{-(\varepsilon_c-\varepsilon_d)\beta}}$$

$$\Rightarrow 1 + \frac{U_c}{2N_D} e^{-(\varepsilon_c-\varepsilon_d)\beta} = \frac{5\times10^{18}}{4.584\times10^{18}}$$

$$\Rightarrow \frac{8\times10^{16}}{2\times5\times10^{18}} e^{-(\varepsilon_c-\varepsilon_d)\beta} = 0.09075$$

$$\Rightarrow e^{-(\varepsilon_c-\varepsilon_d)\beta} = 11.3438$$

$$\Rightarrow -\frac{\varepsilon_c-\varepsilon_d}{k_BT} = 2.4287$$

$$\Rightarrow \varepsilon_c-\varepsilon_d = -\frac{2.4287\times1.38\times10^{-23}\times35}{1.6\times10^{-19}}\ eV$$

$= -7.33$ meV $\equiv$ Donor ionization energy.

b) $N_A = 7.5\times10^{15}\ cm^{-3}$ (of the p side)

$\frac{U_c}{U_v} = 0.08$ @ room temp

$\phi_b = 0.78\ V = \frac{kT}{q} \ln\left(\frac{n_n \cdot p_p}{n_i^2}\right) = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$ at room temp.

$$\Rightarrow 0.78 = 25.875\ meV \times \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$\Rightarrow \ln\left(\frac{N_A N_D}{n_i^2}\right) = 30.14$$

$$\Rightarrow \frac{N_A N_D}{n_i^2} = 1.24\times10^{13}$$

$$\Rightarrow n_i^2 = \frac{7.5\times10^{15}\times5\times10^{18}}{1.24\times10^{13}} = U_c U_v e^{-\Delta\varepsilon_g\cdot\beta}$$

$U_c(T=300K) = U_c(T=35K)\times\left(\frac{300}{35}\right)^{3/2}$

$= 2\times10^{18}$

$U_v(T=300K) = \frac{U_c(T=300K)}{0.08}$

$= 2.51\times10^{19}$

$$\Rightarrow e^{-\Delta\varepsilon_g\cdot\beta} = \frac{7.5\times10^{15}\times5\times10^{18}}{1.24\times10^{13}\times2\times10^{18}\times2.51\times10^{19}} = 6.02\times10^{-17}$$

$$\Rightarrow -\frac{\Delta\varepsilon_g}{k_BT} = -37.35$$

$\therefore \Delta\varepsilon_g \simeq 0.97\ eV$

c) $\frac{n_d}{N_D} = \frac{1}{1 + \frac{u_c}{2N_D} e^{-(\mathcal{E}_c - \mathcal{E}_D)\beta}}$

It is given that,

$\mathcal{E}_c - \mathcal{E}_D = \mathcal{E}_A - \mathcal{E}_v = -7.33$ meV

$\frac{p_a}{N_A} = \frac{1}{1 + \frac{u_v}{2N_A} e^{-(\mathcal{E}_A - \mathcal{E}_v)\beta}}$

$u_v(T=35K) = u_v(T=300K) \times \left(\frac{35}{300}\right)^{3/2}$

$= 1 \times 10^{18}$

$= \frac{1}{1 + \frac{1 \times 10^{18}}{2 \times 7.5 \times 10^{15}} e^{7.33/3.01875}}$

$= 1.32 \times 10^{-3}$

$p_a = 9.91 \times 10^{12}$ cm^{-3}

now, $n_n(T=35K) = N_D - n_d = (5 \times 10^{18} - 4.16 \times 10^{17}) = 4.584 \times 10^{18}$ cm^{-3}

$p_p(T=35K) = N_A - p_a = (7.5 \times 10^{15} - 9.91 \times 10^{12}) = 7.49 \times 10^{15}$ cm^{-3}

$\therefore \phi_b \Big|_{T=35K} = \frac{K_B T}{q} \ln\left(\frac{n_n p_p}{n_i^2}\right) = 3.01875 \times 10^{-3} \ln\left(\frac{4.584 \times 10^{18} \times 7.49 \times 10^{15}}{u_c u_v \exp(-\Delta\mathcal{E}_g \beta)\big|_{T=35K}}\right)$

$= 3.01875 \times 10^{-3} \ln\left[\frac{4.584 \times 10^{18} \times 7.49 \times 10^{15}}{8 \times 10^{16} \times 1 \times 10^{18} \times \exp(-0.97/3.01875 \times 10^{-3})}\right]$

$= 3.01875 \times 10^{-3} \times \{79.52 - 80.37 + 321.33\}$

$= 0.97$ V

4. a) Brillouin function is given by,

$$B_J(y) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} y\right) - \frac{1}{2J} \coth \frac{y}{2J}$$

now for $J = 1/2$,

$$B_{1/2}(y) = 2\coth(2y) - \coth(y)$$

$$= 2\,\frac{e^{2y}+e^{-2y}}{e^{2y}-e^{-2y}} - \frac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$$

$$= \frac{2e^{2y}+2e^{-2y} - e^{2y} - e^{-2y} - 2}{(e^{y}+e^{-y})(e^{y}-e^{-y})}$$

$$= \frac{e^{2y}+e^{-2y} - 2e^{y}e^{-y}}{(e^{y}+e^{-y})(e^{y}-e^{-y})}$$

$$= \frac{(e^{y}-e^{-y})^2}{(e^{y}+e^{-y})(e^{y}-e^{-y})}$$

$$= \frac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$$

$\therefore\ B_{1/2}(y) = \tanh(y)$

b) $$y = \frac{g_J \mu_B J(\cancel{B}^{\,0} + \lambda M)}{k_B T} = \frac{g_J \mu_B J \lambda M}{k_B T}$$

$$= \frac{2 \times \mu_B \times 1/2\, \lambda M}{k_B T}$$

$$y = \frac{\mu_B \lambda M}{k_B T}$$

$J = S = 1/2$

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

$$= 3/2 + \frac{1/2 \times 3/2 - 0}{2 \times 1/2 \times 3/2}$$

$$= 2$$

c) $\frac{M}{M_S} = B_J(y) = \tanh(y)$

$M = M_S B_J(y)$

$M_S = n g_J \mu_B J$

$$T_c = \frac{g_J \mu_B (J+1) \lambda M_S}{3 k_B} = \frac{g_J \mu_B (J+1) \lambda \cdot n g_J \mu_B J}{3 k_B}$$

$$= \frac{2 \times \mu_B \times 3/2 \times \lambda \times n \times 2 \times \mu_B \times 1/2}{3 k_B}$$

$$= \frac{\mu_B^2 n \lambda}{k_B}$$

now, $\tanh(y) = \frac{M}{M_S} = \frac{\frac{k_B T \cdot y}{\lambda \mu_B}}{n g_J \mu_B J}$ | $y = \frac{\mu_B \lambda M}{k_B T} \Rightarrow M = \frac{k_B T}{\lambda \mu_B} y$

$$= \underbrace{\frac{k_B}{\lambda \mu_B^2 \cdot n}}_{= 1/T_c} \cdot T \cdot y$$

$$\therefore \left(\frac{T}{T_c}\right) y = \tanh(y)$$

d) We know, $y = \frac{\mu_B \lambda M}{k_B T}$ $\Rightarrow$ So, when $T \to 0$, $y \to \infty$

$$\tanh(y) = \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{1 - e^{-2y}}{1 + e^{-2y}} \simeq (1 - e^{-2y})^2$$; exponential term is very small as $y \to \infty$

$$\simeq (1 - 2e^{-2y})$$

Now, $\frac{M}{M_S} = B_J(y) = \tanh(y)$

$\Rightarrow M = n g_J \mu_B J \cdot \tanh(y)$

$\Rightarrow M(T \to 0) = n \mu_B (1 - e^{-2y})$; $g_J = 2, J = 1/2$

Also for $T \to 0$ ie, $y \to \infty$ we can write, $y = \frac{T_c}{T}$ considering $\tanh(y)\big|_{y \to \infty} = 1$

$$\therefore M(T \to 0) = n \mu_B (1 - e^{-2T_c/T})$$

5. From the given informations, $T_c = 9K$

London penetration depth varies as, $\lambda(T) = \lambda(0)\left[1-\left(\frac{T}{T_c}\right)^4\right]^{-1/2}$

Now, $\lambda(0) = \sqrt{\frac{m_s}{\mu_0 n_s q_s^2}}$

$m_s = 2m_e$

$n_s = \frac{n_e}{2}$ → electron density

$q_s = 2q_e$

$$= \sqrt{\frac{9.1 \times 10^{-31}}{1.2566 \times 10^{-6} \times 5.5 \times 10^{22} \times (1.6 \times 10^{-19})^2}}$$

$$= 2.27 \times 10^{-5} \text{ cm}$$

$$\therefore \lambda(T = 7.5K) = 2.27 \times 10^{-5}\left[1-\left(\frac{7.5}{9}\right)^4\right]^{-1/2} = 3.15 \times 10^{-5} \text{ cm}$$

Let the magnetic field will decay as, $B = B_s e^{-z/\lambda}$

In the question $\frac{B}{B_s} = \frac{5}{100}$ $\Rightarrow \frac{5}{100} = e^{-z/\lambda}$

$$\Rightarrow \frac{z}{\lambda} = 2.9957$$

$$\Rightarrow z = 2.9957 \times 3.15 \times 10^{-5} \text{ cm}$$

$$\therefore z = 9.44 \times 10^{-5} \text{ cm}$$