PH 569: Applied Solid State Physics (End-Sem)

Course Instructor: S. Mahapatra
24-11-2023, Time 180 min, Total Marks: 40

1. Consider a *2-dimensional* intrinsic semiconductor for which the density-of-states (DOS) functions can be written as

$$g_n(\varepsilon) = (4\pi m_n^*/h^2) \quad \text{for } \varepsilon > \varepsilon_C$$

$$g(\varepsilon) = 0 \quad \text{for } \varepsilon_V < \varepsilon < \varepsilon_C$$

$$g_p(\varepsilon) = \left(4\pi m_p^*/h^2\right) \quad \text{for } \varepsilon < \varepsilon_V$$

How far above the mid-gap is the intrinsic Fermi level at room temperature, if $m_p^*/m_n^* = 2$? (8)

2. The following information is known for a ferromagnetic material.

Crystal Structure	BCC
Lattice parameter	2.8665 Å
Curie point	997.36 K
Weiss parameter (with relates the internal magnetic field B_i with M)	$\lambda = 100\mu_0$
The parameter $g^L J$ (g^L: Lande g-factor and J total angular momentum quantum number)	6

What is the electronic configuration of the partially-filled d-shell of the ferromagnetic element? (*No orbital angular momentum quenching should be assumed*) **(6)**

3. At T = 35 K, the electron concentration of a *n*-type semiconductor with doping density of $N_D = 5 \times 10^{18} \text{cm}^{-3}$ is $n_0(T = 35\ K) = 4.16 \times 10^{17} \text{cm}^{-3}$.

(a) What is the ionization energy of the donor, given $U_C = 2\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} = 8 \times 10^{16}\ \text{cm}^{-3}$ at T = 35 K ?

(*Assume that E_F is several $k_B T$ away from E_D and hence also E_C, so that Boltzmann approximation is valid. Also neglect thermally generated carrier concentration*)

The *n*-type semiconductor of (a) is used to make a *p-n* junction with another *p*-type piece of the same semiconductor, with doping density $N_A = 7.5 \times 10^{15}\ \text{cm}^{-3}$.

(b) At room temperature, if $U_C/U_V = 0.08$, and the built–in potential of the junction is 0.78 V, what is the band gap of the semiconductor?

(c) What is the built-in potential of the same junction at T = 35 K, if the donor and acceptor ionization energies are identical? (*Neglect variation of bandgap with temperature*) (2+ 4 + 3 = 9)

4. For a ferromagnetic material with J = S = ½, show that

(a) Show that $B_J(y) = \tanh(y)$

(b) What is the expression of y in the absence of any externally applied magnetic field?

(c) Show that $\frac{T}{T_C}y = \tanh(y)$ (in the absence of an external magnetic field)

(d) Show that the magnetization at the limit $T \to 0$ is $M(T \to 0) = n\mu_B\left(1 - 2e^{-2T_C/T}\right)$

(2+2+4+4 = 11)

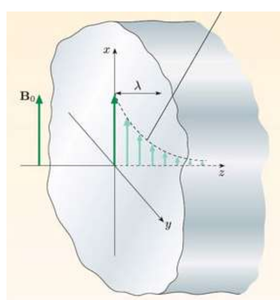
5. The plot of v_k^2 versus electron energy ξ_k (as described in BCS theory) for a superconductor at *T* = 0 resembles the Fermi distribution for a normal metal at *T* = 9 K.

Here, the BCS ground state is being approximated as

$$|\phi_{BCS}\rangle = \prod_k \left[\left(\sqrt{1 - v_k^2}\right)|0\rangle_k + v_k|1\rangle_k\right]$$

where $|0\rangle_k$ and $|1\rangle_k$ represent the states in which the pair state $(\boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow)$ is unoccupied and occupied, respectively.

Given that the London penetration depth varies as $\lambda(T) = \lambda(0)[1 - (T/T_C)^4]^{-1/2}$, estimate the distance (in the *z*-direction) within a semi-infinite slab of this superconductor (in the geometry shown below) over which a magnetic field (of strength less than the critical field of the superconductor at 7.5 K) will decay to 5% of its value at the surface at T = 7.5 K.



(Assume that the electron density of the material in its normal state is $5.5 \times 10^{22} cm^{-3}$ *and consider the free electron mass if needed.)* **(6)**