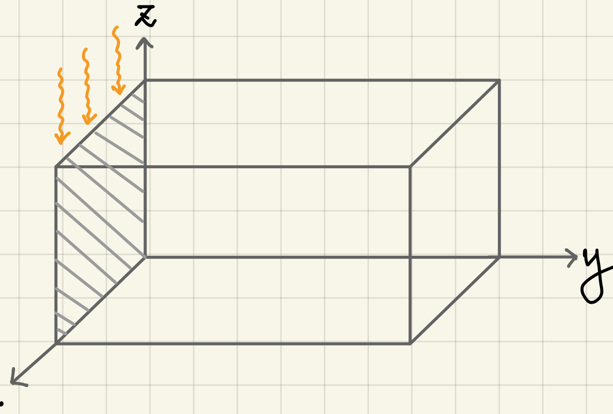
MID SEM Solutions

1. $\frac{d^2}{dy^2}(\delta p) - \frac{\mu_p E_0}{D_p}\frac{d}{dy}(\delta p) - \frac{\delta p}{L_p^2} = 0$; $L_p = \sqrt{D_p \tau_p}$



$$\delta p(y) = p(y) - p_0 = A e^{\nu_p^+ y/L_p} + B e^{\nu_p^- y/L_p}$$

Solving for $\nu_p^\pm$ gives,

$$\nu_p^\pm = \nu_p \pm \sqrt{1+\nu_p^2} \quad ; \quad \nu_p = \frac{\mu_p E_0 L_p}{2 D_p}$$

Here ν_p^+ is >0 and ν_p^- is <0 $\forall\ E_0$

Since $\delta p(y)$ should $\to 0$ as $y \to \pm\infty$

$$\delta p(y) = B e^{\nu_p^- y/L_p} \quad \forall\ y>0$$
$$= A e^{\nu_p^+ y/L_p} \quad \forall\ y<0$$

Also since $\delta p(y)\big|_{y\to 0^+} = \delta p(y)\big|_{y \to 0^-}$

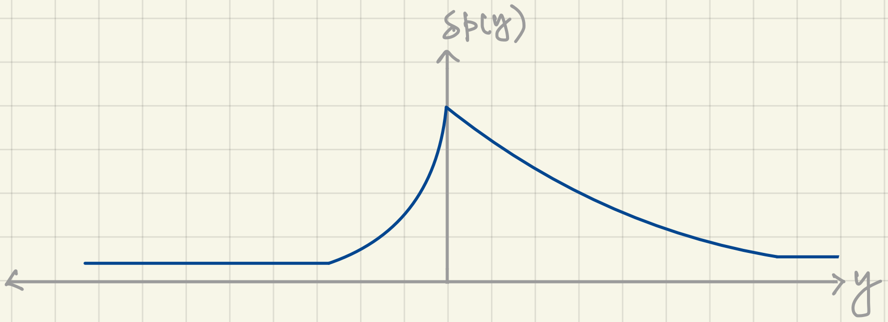
$$A = B = A_0$$

$$\delta p(y) = A_0 e^{\nu_p^- y/L_p} \quad \forall\ y>0$$
$$= A_0 e^{\nu_p^+ y/L_p} \quad \forall\ y<0$$

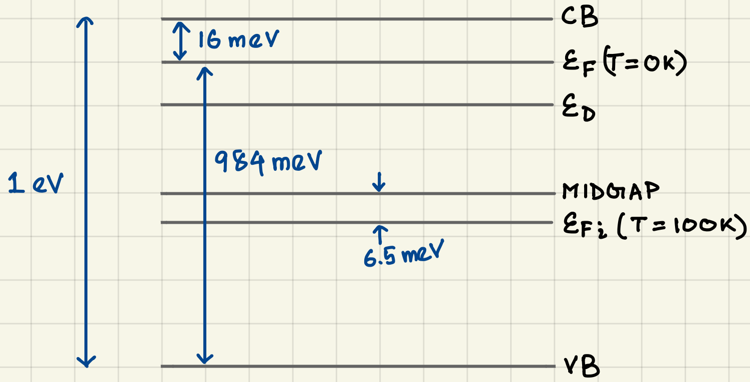
Comparing with the given expressions,

$$\theta = \frac{\nu_p^-}{L_p} \quad ; \quad \lambda = \frac{\nu_p^+}{L_p}$$

$$\therefore\ \theta\lambda = \frac{\nu_p^- \cdot \nu_p^+}{L_p^2} = \frac{\left(\nu_p - \sqrt{1+\nu_p^2}\right)\left(\nu_p + \sqrt{1+\nu_p^2}\right)}{L_p^2} = -1/L_p^2$$



2. At $T = 0K$, the $\mathcal{E}_F$ of a n-type semiconductor lies at $\frac{\mathcal{E}_c + \mathcal{E}_D}{2}$ ie, midway b/w the CB & the Donor level.



$\therefore \mathcal{E}_D = \mathcal{E}_c - 32\ meV$

$\therefore$ Donor binding energy is 32 meV.

$$(13.6\ eV)\ \frac{m_e^*}{k^2} = 32 \times 10^{-3}\ eV$$

$$\Rightarrow m_e^* = \frac{0.032 \times 11^2}{13.6} = 0.285\ m_0$$

At 100K : $\mathcal{E}_{F_i} - \text{midgap} = \frac{3}{4} k_B T \ln\left(\frac{m_h^*}{m_e^*}\right) = -6.5\ meV$

$$\Rightarrow \frac{3}{4} \times 25.7 \times \ln\left(\frac{m_h^*}{m_e^*}\right) = -6.5$$

$$\Rightarrow \ln\left(\frac{m_h^*}{0.285}\right) = -\frac{6.5 \times 4}{3 \times 25.7}$$

$$\Rightarrow m_h^* = 0.2\ m_0$$

$\therefore$ Exciton binding energy,

$$E_{ex} = (13.6) \times \frac{\mu}{k^2}$$

$$= 13.6 \times \frac{0.285 \times 0.2}{0.285 + 0.2} \times \frac{1}{11^2}$$

$$= 13.2\ meV.$$

3. $$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} j_x^{hh} \\ j_y^{hh} \end{pmatrix} + \begin{pmatrix} j_x^{lh} \\ j_y^{lh} \end{pmatrix} = \left\{ \sigma_{hh} \begin{pmatrix} 1 & \omega_c^{hh}\tau_{hh} \\ -\omega_c^{hh}\tau_{hh} & 1 \end{pmatrix} + \sigma_{lh} \begin{pmatrix} 1 & \omega_c^{h}\tau_{lh} \\ -\omega_c^{lh}\tau_{lh} & 1 \end{pmatrix} \right\} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\therefore\ j_x = \left(\sigma_p^{hh} + \sigma_p^{lh}\right) E_x + \left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right) B \cdot E_y$$

$$j_y = -\left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right) B \cdot E_x + \left(\sigma_p^{hh} + \sigma_p^{lh}\right) E_y$$

Now $j_y = 0$ implies,

$$E_x = \frac{\sigma_p^{hh} + \sigma_p^{lh}}{\left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right) B} E_y$$

$$j_x = E_y \left\{ \frac{\left(\sigma_p^{hh} + \sigma_p^{lh}\right)^2}{\left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right) B} + \left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right) B \right\}$$

$$\Rightarrow R_H = \frac{E_y}{j_x B} = \frac{\left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right)}{\left(\sigma_p^{hh} + \sigma_p^{lh}\right)^2 + \cancel{\left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right)^2 B^2}}$$

0

as $\mu_p B \ll 1$

$$= \frac{\left(\sigma_p^{hh}\mu_p^{hh} + \sigma_p^{lh}\mu_p^{lh}\right)}{\left(\sigma_p^{hh} + \sigma_p^{lh}\right)^2}$$

$\sigma = \mu n e$

$\Rightarrow \frac{\sigma}{ne} = \mu$

$\Rightarrow \sigma \cdot R_H = \mu$

$$R_H = \frac{\sigma_p^{hh^2} R_H^{hh} + \sigma_p^{lh^2} R_H^{lh}}{\left(\sigma_p^{hh} + \sigma_p^{lh}\right)^2}$$

4. $\mu_n = \dfrac{e\tau}{m_e^*(\text{cond})} = 600 \times 10^{-4}$

$$\Rightarrow m_e^*(\text{cond}) = \frac{1.6\times10^{-19} \times 86\times10^{-15}}{600\times10^{-4}} = 0.25\, m_0$$

But $\dfrac{1}{m_e^*(\text{cond})} = \dfrac{1}{3}\left(\dfrac{2}{m_\perp^*} + \dfrac{1}{m_\parallel^*}\right)$

Given: $m_\parallel^* = m^*$, $m_\perp^* = 2m^*$

$$\therefore \frac{1}{m_e^*(\text{cond})} = \frac{1}{3}\left(\frac{2}{2m^*} + \frac{1}{m^*}\right) = \frac{2}{3m^*}$$

$$\Rightarrow m^* = \frac{2}{3} m_e^*(\text{cond}) = \frac{2}{3} \times 0.25\, m_0 = 0.166\, m_0$$

$$\therefore m_\parallel^* = 0.166\, m_0$$

$$m_\perp^* = 2 \times 0.166\, m_0 = 0.332\, m_0$$

Intrinsic carrier concentration (@ T = 300K)

$$n_i = \sqrt{N_c N_v}\; e^{-E_g/2k_BT}$$

$$= 2\left(\frac{2\pi\left(m_{P(DOS)}^* m_{n(DOS)}^*\right)^{1/2} K_B T}{h^2}\right)^{3/2} e^{-E_g/2k_BT}$$

- $m_{n\,(DOS)}^{*\,3/2} = \sqrt{m_\perp^2\, m_\parallel}$
  $= \sqrt{(0.332)^2 \times (0.166)}\; m_0^{3/2}$
  $m_n^*(DOS) = 0.2635\, m_0$

- $m_{P\,(DOS)}^{*\,3/2} = m_{hh}^{*\,3/2} + m_{lh}^{*\,3/2}$
  $\Rightarrow m_{P(DOS)}^* = \left\{(0.48)^{3/2} + (0.16)^{3/2}\right\}^{2/3} m_0$
  $= 0.539\, m_0$

$$\therefore n_i = 2\times\left(\frac{2\pi\sqrt{0.539\times0.2635}\times9.1\times10^{-31}\times1.38\times10^{-23}\times300}{(6.626\times10^{-34})^2}\right)^{3/2} e^{-\frac{0.77}{2\times0.0258}}$$

$$= 1.91\times10^{18}\ /m^3.$$

5. $n_0(T=300K) = N_D = 2\times10^{17}\ cm^{-3}$; $m_e^* = 0.4 m_0$; $m_h^* = 0.7 m_0$

$$n_i = \sqrt{u_c u_v}\ e^{-E_g/2k_BT}$$

$$= 2.51\times10^{19} \times \sqrt{(0.4\times0.7)^{3/2}}\ e^{-1100/2\times25.8}$$

$$= 5.33\times10^{9}\ cm^{-3}$$

given, $F_n - F_p = 420\ meV$

Now, $\mathcal{E}_F(T=300K) = \mathcal{E}_{F_i} + k_BT \ln\left(\frac{N_d}{n_i}\right)$

$$= \mathcal{E}_{F_i} + 25.8 \ln\left(\frac{2\times10^{17}}{5.33\times10^{9}}\right)$$

$$= \mathcal{E}_{F_i} + 450\ meV$$

$$\Rightarrow \mathcal{E}_F(T=300K) - \mathcal{E}_{F_i} = 450\ meV.$$

$$\mathcal{E}_{F_i} - F_p = \mathcal{E}_{F_i} - F_p + F_n - F_n = (F_n - F_p) - (F_n - \mathcal{E}_{F_i}) \simeq (F_n - F_p) - (\mathcal{E}_F - \mathcal{E}_{F_i})$$

$$= 420 - 450\ meV$$

$$= -30\ meV.$$

$$\therefore\ \delta p = n_i e^{(\mathcal{E}_{F_i} - F_p)/k_BT} = g\,\tau_p$$

$$\Rightarrow g = \frac{5.33\times10^{9} \times e^{-30/25.8}}{2\times10^{-6}} = 8.33\times10^{14}\ /cm^3.s.$$