PH 569: Applied Solid State Physics (Quiz 2)

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1. A material with similar "effective number of Bohr magnetons", $p = g_L\sqrt{J(J+1)}$ as Dy^{3+} undergoes ferromagnetic transition at $T_C = 1000\ K$. Knowing that the Weiss field H_E is proportional to the magnetization (M) close to T_C (on the paramagnetic side), find the proportionality constant λ (molecular field parameter), if the number density of ions is $6x10^{27}m^{-3}$. **(3 marks)**

2. Prove that

$$\langle\varphi_k|\boldsymbol{S}_{xi}\boldsymbol{S}_{xj} + \boldsymbol{S}_{yi}\boldsymbol{S}_{yj}|\varphi_k\rangle = \frac{2S}{N}\cos\left[\boldsymbol{k}.\left(\boldsymbol{R}_i - \boldsymbol{R}_j\right)\right]$$

where,

$$|\varphi_k\rangle = \frac{1}{\sqrt{N}}\sum_j e^{i\boldsymbol{k}.\boldsymbol{R}_j}|j\rangle$$

and

$$|j\rangle = |S,S\rangle_1|S,S\rangle_2|S,S\rangle_3 \ldots .|S,S-1\rangle_j \ldots.$$

(4 Marks)

3. Consider a 1D spin chain, where the interaction is described by the Hamiltonian

$$H_{MG} = -K\sum_p\left(\frac{4}{3}\boldsymbol{S}_p.\boldsymbol{S}_{p+1} + \frac{2}{3}\boldsymbol{S}_p.\boldsymbol{S}_{p+2}\right)$$

with $K < 0$.

Notice that the 2nd nearest-neighbour interaction has been considered, which is weaker than the nearest-neighbour interaction, but both are antiferromagnetic.

Show that

$$|\pm\rangle = \frac{1}{\sqrt{2}}\prod_{i=1}^{N/2}\left(|S_{2i},S_{2i}\rangle|S_{2i\pm1},-S_{2i\pm1}\rangle - |S_{2i},-S_{2i}\rangle|S_{2i\pm1},S_{2i\pm1}\rangle\right)$$

is an eigenstate of H_{MG}.

(Hint: Define the operator $O_i = \frac{1}{3}\left(\boldsymbol{J}_i.\boldsymbol{J}_i - \frac{3}{4}\right)$, where $\boldsymbol{J}_i = \boldsymbol{S}_{i-1} + \boldsymbol{S}_i + \boldsymbol{S}_{i+1}$) and find the eigen values of O_i for $J_i = \frac{1}{2}$ and $\frac{3}{2}$. Then write O_i in terms of $\boldsymbol{S}_i$ and try to identify H_{MG}).

(*This is known as the Majumdar-Ghosh Hamiltonian*, in which "Ghosh" stands for Prof. Dipan K Ghosh of the Physics Department, IIT Bombay) **(8 marks)**